# INVESTIGATION OF THE DYNAMICS OF THE DEVELOPMENT OF CRACKS BY THE METHOD OF PHOTOELASTICITY

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An experimental investigation was made of the process of the development of cracks in two materials: polymethyl methacrylate and a polymerized epoxy resin. For these materials, determinations were made of their optical and mechanical characteristics, such as density, the speed of sound, the critical value of the coefficient of singularity of the stress field at the tip of cracks. As the dynamic characteristic of the process of the development of cracks, an investigation is made of the dependences connecting the coefficient of the singularity of the stress field at the tip of a moving crack and the rate of development of the crack. The question of the interaction between two cracks moving in a previously elongated sample is discussed.

Methods of photoelasticity are widely used for investigating the static and dynamic fields of the stresses and deformations in elastic bodies. Specifically, these methods are used to study the process of the development of cracks [1, 2].

The dependences connecting the coefficients of singularity of the field of the stresses at the tip of a moving crack and the rate of development of the crack close the system of the equations of motion of the crack [3]; therefore, their experimental investigation is very important.

## 1. Experimental Method

The experiments were made using the usual scheme of a polarization-optical unit [4], illustrated arbitrarily in Fig. 1. Its main parts were the illuminating device 1, the polaroids 2, the four-wave plates 3, the fracture device 5, and the objectives 6.

In static tests, an incandescent lamp was used in the illuminating device, and, with dynamic tests, an IFP-1000-2A pulsed lamp. The objectives made it possible to create a parallel beam of light with a diameter of 100 mm in the working section. The light passed through a Ba55J interference filter at 557 nm.

Depending on the character of the experiment, the photographic device was a "Pentazet" camera, a "Konvas" moving-picture camera, or a high-speed streak camera. The fracture device made it possible to statically elongate rectangular flat samples; under these circumstances, it was possible to record the elongating forces and the elongation of the sample. The clamping device ensured clamping of the sample along the whole length of the clamped edges without misalignment, which was verified on samples with symmetrically arranged notches (Fig. 2a).

With elongation of the sample, the skew of the movable clamp was recorded. The experiments described in the present article were carried out with a zero skew of the clamp.

In dynamic tests, a crack in a previously elongated sample was initiated using an electric striker, setting into motion a knife, which was applied to the sample. The construction of the electric striker was analogous to that described in [5]. The striker operated from the discharge of a battery of condensers (12 μF, 25 kV), synchronized with a streak camera. In experiments where the motion of two cracks was recorded, two electric strikers

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were used, connected in series. In this case, a lag in the development of one of the cracks could be created by regulating the gap between the knife and the edge of the sample. The IFP-1000-2A lamp worked off a battery of condensers with a capacitance of 700 μF and a voltage of 1.5 kV. The lamps were ignited from the delayed pulse of the streak camera. A G5-15 generator was used for the delay.

The experiments were made using rectangular samples measuring 50 × 150 mm². The thickness of the samples varied within the limits 2.8-4.5 mm. The geometry of the notches or cracks was different in different series. Figure 2 shows the configurations used in the work. The tests were made on samples made of sheet polymethyl methacrylate and a hardened epoxy resin. The plates made of the epoxy resin were made by the polymerization of ÉD-5 with maleic anhydride. The molds were glass plates, treated with dimethylchlorosilane using the method of the Laboratory for the Investigation of Stresses of the V. V. Kuybyshev Construction Engineering Institute, Moscow.

The mechanical and optical properties of the materials used are given in Table 1, where $\rho$ is the density; $C_p$, $C_s$ are the velocities of the longitudinal waves in the plate; $\nu$ is the Poisson coefficient; $E_s$, $E_d$ are the static and dynamic Young coefficients; $\sigma_0$ is the static value of the optical constant of the material. The values of $E_{d,\nu}$ were determined from the values of $\rho$, $C_p$, $C_s$, which were found from moving-picture photos of the propagation of waves from the explosion of a charge of tin azide, arranged at the boundaries of the plate. The table gives the mean values of the above quantities; deviations from these values may reach 10% in actual samples.

## 2. Method for Analyzing the Picture of the Isochromatic Curves

An important factor in the application of the method of photoelasticity to the investigation of the dynamics and statics of cracks is the question of interpretation of the picture of the isochromatic curves at the tip of a crack. The most exact method for such an interpretation is given in [2]. In this article, it is assumed that the field of the stresses near the tip of a crack is represented in the form

$$\sigma_{ik} = \frac{N}{\pi\sqrt{r}} f_{ik}(v, C_p, C_s, \vartheta, r) + F_{ik}(v, C_p, C_s, \vartheta, r),\ f_{22} = 1 \text{ with } \vartheta = 0,$$

where i, k = 1, 2 (the axis 2 is perpendicular to the line of the crack); v is the velocity of the crack; r, $\vartheta$ are the coordinates of a polar system of coordinates with its origin at the tip of the crack; N are coefficients of the singularity of the field of the stresses; $f_{ik}$, $F_{ik}$ are functions not having a singularity with respect to r with $r \to 0$.

The form of the functions $f_{ik}(r, \vartheta)$, $F_{ik}(r, \vartheta)$ is assumed to be the same as in statics. In this case, use is made of an overall representation of the stressed state near the tip of a notch in the form of series in terms of r [6]. The investigations of the authors of [2] showed that completely satisfactory accuracy in determination of the coefficient of singularity of the field of the stresses N is obtained using only the zero terms in the expansions of $f_{ik}$, $F_{ik}$ in terms of r. In this case, for the maximal tangential stress $\tau$ we obtain the following expression:

$$\tau^2 = \frac{1}{4}\left(\frac{N^2}{\pi^2 r}\sin^2\vartheta + \frac{2N\sigma}{\pi\sqrt{r}}\sin\vartheta\sin\frac{3}{2}\vartheta + \sigma^2\right)$$

The parameters N and $\sigma$ must be so selected that the isochromatic curves will best coincide with the experimental curves. Previn [1] proposed finding N and $\sigma$ from the coordinates of the furthest removed point of some loop of the isochromatic curve. A shortcoming of this method is the fact that it is very limited with respect to accuracy in determination of the angular coordinate of the selected point. In the present work the parameters N and $\sigma$ were determined from the dimensions of the radii of a pair of loops in a direction perpendicular to the line of the crack.

In this case, the formula for determining the value of N has the following form:

$$N = 2\tau_1\sqrt{r_1}\,\delta\pi$$

$$\delta^2 = \frac{1}{1+\alpha}\left\{\frac{(1-\beta)\sqrt{\alpha}}{1-\sqrt{\alpha}} + 1 - \left[1 + \frac{2(1-\beta)\sqrt{\alpha}}{1-\sqrt{\alpha}} - \frac{1-\beta}{(1-\sqrt{\alpha})^2}\right]^{1/2}\right\}$$

$$\alpha = r_1 / r_2,\ \beta = \tau_2^{\,2} / \tau_1^{\,2}$$

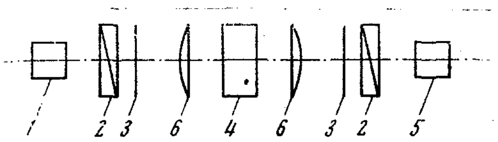

Fig. 1

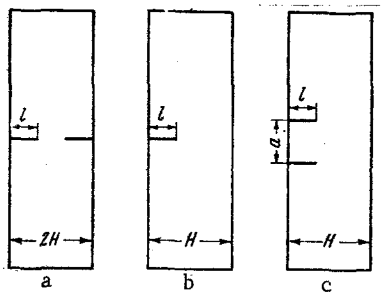

Fig. 2

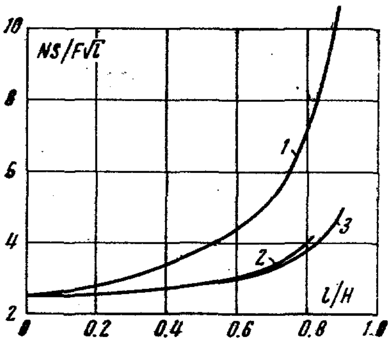

Fig. 3

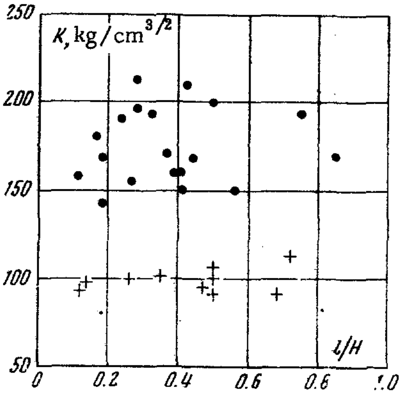

Fig. 4

Here $\tau_1$, $\tau_2$ are the values of the stresses for the selected isochromatic curves; $r_1$, $r_2$ are the vertical radii of these isochromatic curves.

If the difference $\beta - \alpha$ is small, the reduced expression for $\delta$ is approximately represented in the form

$$\delta^2 = 1 - q + \alpha^{-1/2}q^2 / 2, \quad q = (\beta - \alpha)\,\alpha^{-1/2}\,(1 - \alpha^{1/2})^{-1}$$

In the analysis of the experimental isochromatic curves in the present work, the isochromatic curve with the smallest dimensions and several neighboring curves were usually selected. The values of the coefficient N obtained for the selected pairs were averaged. The deviation from the mean value generally did not exceed 10%.

## 3. Static Tests

For the materials under investigation, a series of experiments was made with the aim of determining the critical value of K, the coefficient of singularity of the field of the stresses, at which a crack developed with static loading. In the experiments, the stress with which fracture of the sample occurred was recorded. The critical value of the K coefficient of the singularity of the field of the stresses for samples made of epoxy resin was determined from the picture of the isochromatic curves recorded using moving-picture photography at the moment before fracture. For samples made of polymethyl methacrylate, the value of the coefficient K was determined from the value of the critical stress and the dependence of the reduced coefficient of singularity $Ns/F\sqrt{l}$ on the reduced length of the crack $l/H$ for a given geometry of the sample and given loading conditions (Fig. 3, curve 1). Here F is the force of the stress; $l$ is the length of the crack; s is the area of the transverse cross section of the plate; H is the width of the sample. The above dependence was determined experimentally in samples made of epoxy resin. Under these circumstances, the crack was modelled by a thin notch. The picture of the isochromatic curves was photographed for each value of the length of the crack for different loadings. The reduced values found for the coefficient of singularity were averaged.

Figure 4 gives plots of the value of the critical coefficient K of the singularity of the field of the stresses, recorded in experiments on the elongation of samples with a crack, made of polymethyl methacrylate (points) and epoxy resin (crosses).

The considerable scatter of the values of the critical coefficients, particularly for polymethyl methacrylate, can be explained by the fact that the method selected for deter-

TABLE 1

| Material | $\rho$, g/cm$^3$ | $c_p$, m/sec | $c_s$, m/sec | $\nu$ | $E_s$, kg/cm | $E_d$, kg/cm$^2$ | $g_0$ (kg/cm$^2$)(cm/bands) |
|---|---|---|---|---|---|---|---|
| Polymethyl methacrylate | 1.2 | 2250 | 1310 | 0.32 | $3.4\cdot10^4$ | $5.4\cdot10^4$ | — |
| ÉD-5 | 1.23 | 2290 | 1250 | 0.4 | $3.5\cdot10^4$ | $5.4\cdot10^4$ | 14 |

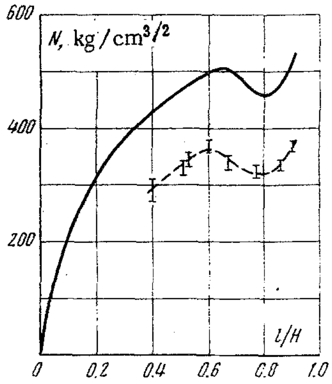

Fig. 5

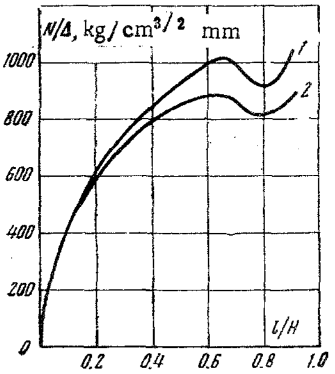

Fig. 6

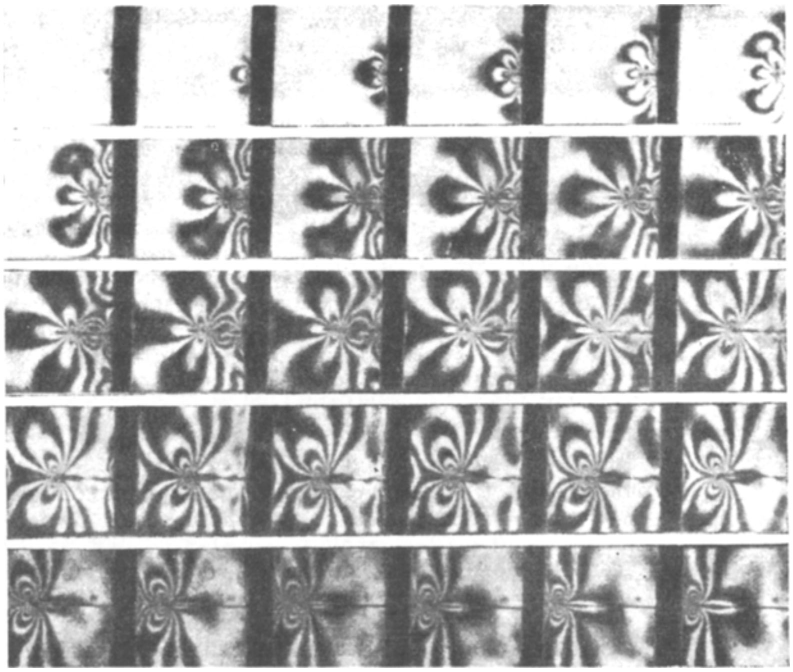

Fig. 7

mining the critical coefficient is very sensitive to the imperfections of a previously induced crack. It has been noted that any given deviation of the initial crack from a plane perpendicular to the surface of the sample and its lateral edge leads to an increase in the critical coefficient. On the basis of this, as the most probable values of the critical coefficient, for polymethyl methacrylate we take K = 150 kg/cm$^{3/2}$, and for epoxy resin, K = 100 kg/cm. The constancy of the critical coefficient K with a change in the length of the initial crack in a sample is evidence of the fact that the materials being studied satisfy the hypotheses of the Griffiths theory of brittle fracture.

Curve 2 in Fig. 3 corresponds to the dependence of $Ns/F\sqrt{l}$ on $l/H$ for a sample with two external symmetrically arranged cracks (Fig. 2a, where H is the half-width of the sample). This type of geometry of the sample and the means used in setting up the stress correspond approximately to the statement of the Bowie problem [7] of the elongation of a

Fig. 8

rectangular sample with two symmetrically arranged lateral notches. Loading in the Bowie problem is effected by a constant elongation normal stress applied to opposite edges of the sample. Bowie solutions of this problem (curve 3 in Fig. 3) are in good agreement with the experimentally found dependence.

Experiments with a moving crack were made with a fixed elongation of the sample. It is interesting to compare the dynamic stresses arising under these circumstances with the static stresses existing with the same elongation of a sample having a notch of corresponding length. To make such a comparison for the coefficients of singularity N of the field of the stresses, the dependence of $N/\Delta$ on $l/H$ ($\Delta$ is the elongation of the sample) was plotted from static experiments. This dependence is shown in Fig. 5; curve 1 corresponds to a scheme with one lateral crack (Fig. 2b), curve 2, to a scheme with two lateral cracks (Fig. 2a).

## 4. Dynamic Tests

A streak camera was used to make moving-picture photos of a crack moving through a previously elongated sample, and of the picture of the isochromatic curves arising in this case in samples of epoxy resin. Experiments were made with one (Fig. 2b) and two (Fig. 2c) lateral cracks; the value of the preliminary elongation of the samples was varied.

The moving-picture photos of the isochromatic curves were used to calculate the singularity coefficient of the field of the stresses, using the method described above. Figure 6 shows the dependence of the coefficient of singularity of the field of the stresses on the length of the crack for a moving crack (curve 1) and the corresponding static dependence (curve 2). The preliminary elongation of all the samples of epoxy resin in this case was 0.5 mm.

This case corresponds to the moving-picture photo of the fields of the isochromatic curves shown in Fig. 7 (dynamic). For purposes of comparison Fig. 8 shows a photo obtained in static tests with $\Delta = 0.3$ mm. Figure 9 gives dynamic (curve 1) and static (curve 2) dependences of the coefficient of singularity of the field of the stresses on the length of the crack, obtained in experiments carried out according to the scheme of Fig. 2c, with two lateral cracks in samples of epoxy resin.

The absence of experimental points on Figs. 6 and 9 with small values of $l/H$ is explained by the fact that the crack in the sample in dynamic experiments was initiated by the impact of the knife at the edge of the sample; therefore, during the initial period of the development of the crack the knife which generated the crack has a strong effect on the picture of the isochromatic curves.

From the curves given in Figs. 6 and 9, it can be seen that, with $l/H > 0.3$, the dynamic coefficient of the singularity of the field of the stresses depends only slightly on the length of the crack and its value in this interval is 30% less than the static value.

For a semi-infinite crack, moving in a previously stressed elastic body, it is shown in [8] that the ratio of the dynamic coefficient of singularity $N_d$ to the static $N_s$ is a completely determined function of the velocity of the crack and of the elastic constants of the body

$$N_d = N_s k (v / C_R, \nu)$$

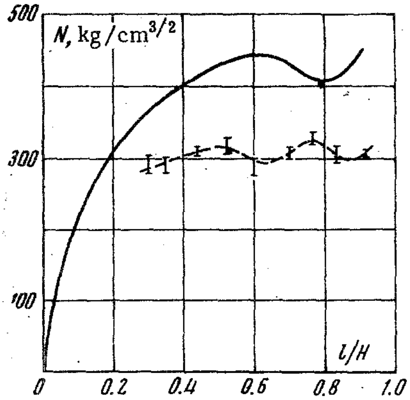

Fig. 9

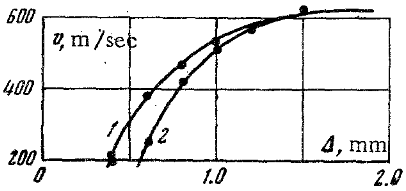

Fig. 10

Here $C_R$ is the velocity of the Rayleigh waves. Measurements of the rate of growth of a crack in samples of epoxy resin showed that, over a wide range of change in the initial elongation, $v = 400$ m/sec, $v/C_R = 0.4$. According to the data of [8], $k(0.4, 0.3) = 0.7$, which is agreement with the values of the dependence illustrated in Figs. 6 and 9.

Moving-picture photos of the fracture were used to determine the dependence of the length and the velocity of the crack on the time. A common feature of cracks in samples of polymethyl methacrylate was the fact that, at the start of their motion, they were accelerated, but then reached conditions with a constant rate and, in the second half of their path, moved uniformly. Figure 10 shows the dependence of the value of the steady-state velocity of a crack on the value of the previous elongation. Curve 1 in Fig. 10 corresponds to experiments in accordance with a scheme with one crack, and curve 2, to a scheme with two cracks.

In samples of epoxy resin, the rate of motion of cracks was found to be constant and equal to 400 m/sec, with a scatter not exceeding 5%.

The dependence of the velocity of a crack on the coefficient of singularity for samples of epoxy resin (curve 1) and polymethyl methacrylate (curve 2) is shown in Fig. 11. Here, along the axis there are plotted the mean rate of motion of a crack in the interval $l/H > 0.5$ and the mean value of $N_d$. The value of $N_d$ for samples of epoxy resin was determined directly from the picture of the isochromatic curves. For samples of polymethyl methacrylate, $N_d$ was determined using a reduced formula; here the value of the parameter $v/C_R$ was determined from experiment, and the value of $N_s$, from the curve of Fig. 5. The small circles in Fig. 11 correspond to experiments with one crack, and the triangles to experiments with two cracks.

All the points (Fig. 11) for each material are grouped around its own curve, with a scatter not exceeding 20%. At the same time, it can be noted that the materials investigated differ very strongly one from the other.

In experiments with a moving pair of cracks, both cracks did not always arrive at the opposite side of the sample. In some cases, due to a random lag, one of the cracks started to develop later, moved at a lower velocity, and remained within the sample. It was found that the fate of the delayed crack depends on the length $l_{10}$ of the crack appearing first, at the moment of the origin of the second crack.

Figure 12 gives the points of the experimental dependence of the final length $l_{2k}/H$ of a delayed crack on $a/l_{10}$ ($a$ is the distance between the points of origin of the cracks). The dimensions of the samples in this series were $100 \times 180$ mm²; $a = 50$ mm; the material was polymethyl methacrylate. It can be seen from the curve of Fig. 12 that, with $a/l_{10} < 2$, the delayed crack does not develop. An evaluation can be made of the degree of effect of a crack on a neighboring one, using the solution of the static problem of an isolated crack with a length of $2l_{10}$ in the field of a homogeneous elongational stress P, acting perpendicular to the line of the crack. The solid curve (Fig. 12) is the dependence of the value $\sigma_{yy}/P$ of the stress at some point on the axis of symmetry of such a curve on the distance to this point, $a$. A sharp drop in the curve with a decrease in $a/l_{10}$ starts in the immediate vicinity of the value $a/l_{10} = 2$.

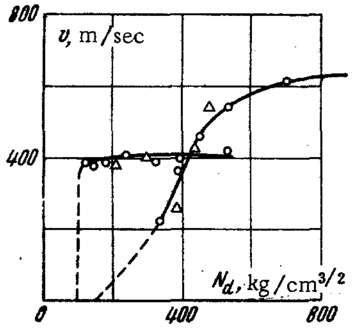

Fig. 11

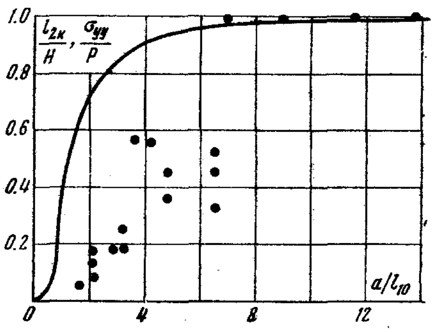

Fig. 12

With $a/l_{10} \geq 7$, in experiments with two cracks, the velocities of the cracks equalized out and both cracks arrived at the opposite edge of the sample. It was remarked that, with a decrease in the value of the preliminary stress, the edge point of this zone was shifted to the right.

Thus, a comparison between static and dynamic stress fields, arising with the passage of a crack through a previously elongated sample of epoxy resin, confirmed the conclusion of [1, 2] that these fields are much alike. Under these circumstances, the ratio of the dynamic coefficient of singularity of the field of the stresses to the static coefficient was found equal to 0.7, which is close to the theoretical value for a semi-infinite crack, obtained in [8].

For polymethyl methacrylate and a polymerized epoxy resin, dependences were found connecting the dynamic coefficient of singularity of the field of the stresses and the velocity of the crack. These dependences were found from experiments carried out in accordance with two different schemes (b and c in Fig. 2), which provide a basis for postulating their universality for the materials studied.

An investigation of the process of fracture, arising with the introduction of two cracks into a previously elongated sample showed that, in a certain range of the scatter of the initial lengths of the cracks originating, both cracks cut through the whole sample. With a large degree of scatter, one of the cracks remains within the sample, and, finally, with some value of the difference in the lengths arising, the smaller crack does not develop at all.

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